Course Readiness

Prerequisite self-check for Intro to Number Theory 2

If you are comfortable with divisibility, prime factorization, divisors, GCD and LCM, remainders, the Euclidean Algorithm, and basic linear Diophantine equations, then you are ready to begin **Intro to Number Theory 2**. Answers appear on the final page.

1. Determine which of $2, 3, 6$, and 9 divide 438.

2. Write 756 as a product of primes.

3. How many positive divisors does 360 have?

4. Find the sum of all positive divisors of 36.

5. Find $\gcd(84, 126)$ and $\operatorname{lcm}(84, 126)$.

6. Write 527 in the form

$$527 = 23q + r, \qquad 0 \le r < 23.$$

7. When an integer n is divided by 8, the remainder is 5. What is the remainder when $3n + 7$ is divided by 8?

8. Use the Euclidean Algorithm to find $\gcd(414, 662)$.

9. Find one integer solution (x, y) of

$$18x + 30y = 6.$$

10. Find all pairs of nonnegative integers (x, y) satisfying

$$4x + 7y = 39.$$

Answer Key

The answers below are intentionally brief. In the course, class discussions and problem sets will emphasize clear reasoning and complete solutions.

1. 438 is divisible by 2, 3, and 6, but not by 9.

2. $756 = 2^2 \cdot 3^3 \cdot 7$.

3. $360 = 2^3 \cdot 3^2 \cdot 5$, so the number of positive divisors is

$$(3+1)(2+1)(1+1) = 24.$$

4. $36 = 2^2 \cdot 3^2$, so the sum of its positive divisors is

$$(1+2+4)(1+3+9) = 91.$$

5. $\gcd(84, 126) = 42$ and $\operatorname{lcm}(84, 126) = 252$.

6. $527 = 23 \cdot 22 + 21$, so $q = 22$ and $r = 21$.

7. The remainder is 6, since $3 \cdot 5 + 7 = 22$.

8. $\gcd(414, 662) = 2$.

9. One solution is $(x, y) = (2, -1)$, because

$$18(2) + 30(-1) = 6.$$

10. The nonnegative solutions are

$$(x, y) = (8, 1) \quad \text{and} \quad (1, 5).$$

What is not required before the course?

You do not need prior knowledge of modular inverses, the Chinese Remainder Theorem, Fermat's Little Theorem, Euler's Theorem, Euler's phi function, or number bases. These ideas are taught in Intro to Number Theory 2.